# DERIVE A NEW DRAG FORMULA ON POROUS MEDIA LAMINAR FLOW FROM THE MINOR RESISTANCE ASSUMPTION

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*Darcy's law seems to end the laminar flow problem of porous media. However, in recent years, many scholars have found that the resistance and velocity of Darcy's law are nonlinear, indicating that the laminar flow mechanism of porous media is still unclear. Further research is necessary and urgent. Based on the shortcomings of the traditional tube flow model, we consider the seepage resistance of porous media as the sum of the numerous average minor resistances and theoretically derive the new resistance formula for laminar flow. Using the experimental data of Darcy, Charles Ritter, and Bağcı et al., the average local resistance coefficient of the porous medium was determined to be 200. Compared with the classical Kozeny–Carman equation and the Ergun equation, the new equation has the best consistency and the least error in all experiment data predictions. However, because the porous media is complex and highly nonlinear, our equation and coefficient still need lots of experimental data and other ways to validate the results.*

**KEY WORDS:** *porous medium, viscous dissipation, conduction limit, channel partially filled with a porous medium*

## 1. INTRODUCTION

Henry Darcy, a French hydraulic engineer, put forward Darcy's law through a paper (1856) on large number of sand seepage experiments, which laid the theoretical foundation of porous media and was widely used in engineering. However, due to the randomness of the porous media structure and the nonlinearity of the flow, researchers have long found that Darcy's law can only be applied to porous media flow with slow velocity (Re $<$ 1).

Dupuit (1863) revised Darcy's law, for the first time considering the difference between apparent velocity and actual velocity. Later, Slichter (1902), Terzaghi (1925), and Darapsky and Müller (1915) began to introduce viscosity and capillary diameter, focusing on the effect of porosity. However, it was not until Blake (1922) adopted dimensionless criteria and semi-empirical analysis that a better correlation was obtained, as shown in Eq. (1):

$$v = \frac{\varepsilon^3}{k\mu S^2}\frac{\Delta P g}{L} \tag{1}$$

where $\Delta P$ is the pressure drop (Pa), $v$ is the superficial or "empty-tower" velocity (m/s), $\varepsilon$ is the porosity of the bed, $k$ is the shape coefficient of the cross-section of the channel, $\mu$ is the viscosity of the fluid (Pa.s), $S$ is the particle surface area ($m^2$), $L$ is the total height of the bed, and $g$ is gravitational acceleration ($m/s^2$).

Based on this, Kozeny and Carman (1938) obtained the widely used Kozeny–Carman equations as shown in Eq. (2) by considering factors such as specific surface area and tortuosity to further refine Eq. (1):

$$\frac{\Delta P}{L} = \frac{180\mu}{\Phi_s^2 D_p^2} \frac{1 - \varepsilon^2}{\varepsilon^3} v \tag{2}$$

where $\Phi_s$ is the sphericity of the particles in the packed bed and $D_p$ is the diameter of the volume equivalent spherical particle (m).

Equation (2) is suitable for laminar flow in porous media with Reynolds number less than 1, and has been widely used in many fields. Chapuis and Aubertin (2003) deeply analyzed a large number of data and found that Kozeny–Carman equation is correct for most soils. The problem arises in the case of anisotropic or unsaturated porous media, or when the specific surface area of porous media is not accurately measured. The tortuosity term of Kozeny–Carman equation was handled by Nooruddin and Hossain (2012) in a more robust manner, and the equation demonstrated its global applicability and significant improvement in identifying a hydraulic flow unit (HFU).

Wan et al. (2013) analyzed the experimental data of Darcy in that year and found that the permeability coefficient $K$ of porous media gradually decreased with the increase of Reynolds number and the permeability coefficient $K$ was not equal to a constant. That is to say, Darcy's seepage experiment did not obey the linear seepage law. Therefore, Wan et al. (2013) was further confirmed by his own experiments. Shi (2017) revised Darcys law based on the seepage equation of soft soil and established a seepage model for low-permeability soft soil considering pore size distribution. Zheng et al. (2017) and Wang et al. (2017) deduced new resistance formulas from the traditional and fractal theory, respectively, based on the new pore throat model of staggered stacking porous media. It was found that the Ergun equation and the new equation had larger prediction errors under the condition of low Reynolds number flow in porous media.

In recent years, with the development of computer software and hardware computational fluid dynamics (CFD) are more and more convenient and become a useful way of conducting research. Zeidan (Zeidan et al., 2019; Zeidan, 2011a) numerically studied the two-dimensional two-phase flow of gas-liquid mixture using a mixed model. Zeidan (2016, 2011b) also systematically evaluated model equations and numerical methods through a series of numerical experiments. Especially Zeidan et al. (2007) proposed a new model and solution method for two-phase compressible flow. The characteristics of the model include the existence of real eigenvalues and a complete set of independent eigenvectors. Peshkov and Romenski (2016) discussed a pure hyperbolic transformation of the parabolic Navier–Stokes equation. It is proved that Newtons law of viscosity can be obtained as the steady-state limit under the framework of hyperbolic theory.

In summary, for porous media, even in the case of laminar flow, its resistance law and characteristics are still not well understood, and further research is still necessary. In this paper we propose a minor resistance model based pore throat unit, which has usually been used to study the resistance of porous media in recent years. The laminar flow resistance loss is assumed as completely caused by local loss according the minor resistance model, the calculation formula of the laminar flow resistance of the porous medium is obtained, and the average local resistance loss coefficient is determined.

## 2. THEORETICAL ANALYSIS OF LAMINAR RESISTANCE IN POROUS MEDIA

### 2.1 Local Resistance Model for Porous Media

#### *2.1.1 Porous Media Characteristic Unit*

Most of the porous media in nature are random and highly complex structures, but they can be considered isotropically in statistical terms. Therefore, it can be assumed that such a porous medium is formed by stacking spherical particles of equivalent diameter ($D_p$), and the characteristic structural unit adopts a well-aligned pore-throat model, which is currently widely used. Its structure is shown in Fig. 1, and the main flow direction of the fluid is shown by the arrow in the figure. Assuming that the contact between the particles is relatively tight, the structural unit length $l$ is approximately equal to the average particle diameter $D_p$.

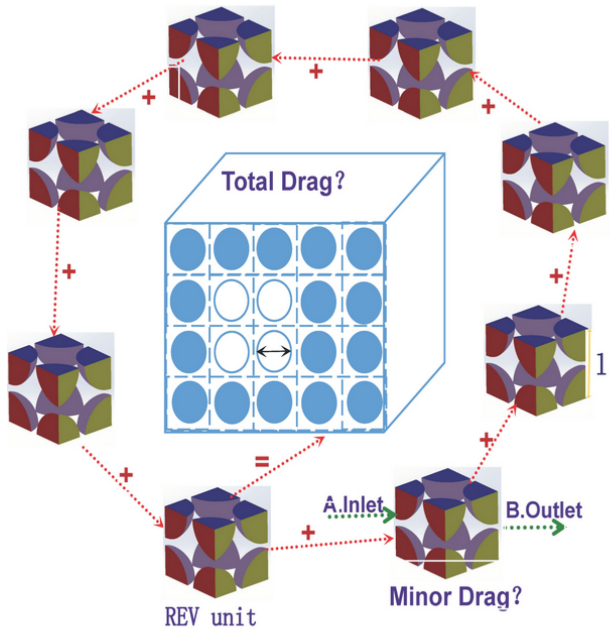



**FIG. 1:** Pore throat characteristic unit

### *2.1.2 Local Resistance Model of Porous Media*

Engels analyzed the disappearance of mechanical motion in the dialectics of nature. He pointed out that friction and collision are essentially the same, but only different in degree. Friction can be seen as a series of small collisions, and collisions can be seen as a violent friction. This statement actually points out that the essence of viscous resistance and inertial resistance is the same. Therefore, the resistance of the porous medium is completely regarded as the minor resistance loss. Based on the structural model of Fig. 2, the fluid in the aligned model flows from A to B and passes through a sudden expansion and a sudden contraction. The drag coefficient of the sudden expansion and contraction in the laminar flow is inversely proportional to the Reynolds number (Re), but the proportional coefficient is different and not defined. Here we think that the resistance of the porous medium is composed of the two average minor losses, its inverse proportional coefficient is denoted with letter $E$, and the average minor loss coefficient $\zeta$ is as follows:

$$\zeta = \frac{E}{\mathrm{Re}} \tag{3}$$

## 2.2 Derivation of Laminar Flow Resistance Formula for Porous Media

The porous media resistance is derived through the following ways. Based on pore throat unit and minor drag model, the pressure drop is got from the fluid dynamics. According to the average hydraulic radius model, the equivalent diameter in the Reynolds number is obtained. The average velocity is expressed in terms of apparent velocity and

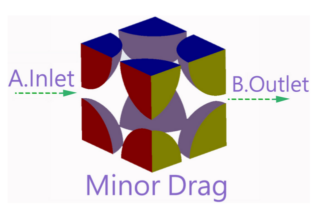



**FIG. 2:** Local resistance model

porosity. Substitute the equivalent diameter and the apparent velocity into the pressure drop. The pressure drop divided by the length of pore throat unit is the resistance of porous media.

According to the assumption of minor resistance, the total resistance loss in the characteristic unit of porous media is equal to the sum of all minor resistance losses, that is

$$\Delta P = \sum_{i=1}^{n} \zeta_i \frac{\rho \bar{v}_i^2}{2} = \zeta_1 \frac{\rho \bar{v}_1^2}{2} + \zeta_2 \frac{\rho \bar{v}_2^2}{2} \tag{4}$$

where $\rho$ is the density of fluid (kg/m$^3$) and $i$ is the number of the minor loss.

According to the aligned pore throat model, the minor loss of the characteristic unit is composed of the sudden expansion (the inlet) and the sudden contraction (the outlet), which is equal to the sum of two average local losses from the assumption ($\zeta_1 = \zeta_2 = \zeta$). Therefore, Eq. (3) is substituted for Eq. (4) to obtain Eq. (5).

$$\Delta P = \zeta_1 \frac{\rho \bar{v}_1^2}{2} + \zeta_2 \frac{\rho \bar{v}_2^2}{2g} = 2 \frac{E}{\mathrm{Re}} \frac{\rho \bar{v}^2}{2} \tag{5}$$

Further derive to draw

$$\Delta P = 2 \frac{E}{(\rho \bar{v} d)/\mu} \cdot \frac{\rho \bar{v}^2}{2} = \frac{E \mu \bar{v}}{d} \tag{6}$$

According to the average hydraulic radius model, the equivalent diameter $d$ of the channel of porous media is as follows:

$$\begin{aligned} d = 4R &= 4 \frac{\text{Effective flow cross-sectional area}}{\text{Wetted perimeter}} = \frac{\text{Effective flow volume}}{\text{Total wetted surface area}} \\ &= 4 \frac{\text{Void volume}}{\text{Bed volume}} \Big/ \frac{\text{Wetted area}}{\text{Bed volume}} = 4 \frac{\varepsilon}{a'} \end{aligned} \tag{7}$$

where $a'$ is bed wetted specific area (m$^2$/m$^3$).

The bed wetted specific area $a'$ can be expressed by the specific area of particles $a_v$ and porosity in the bed. The relationship between them is as follows:

$$\begin{aligned} a' &= \frac{\text{Wetted surface area}}{\text{Bed volume}} = \frac{\text{Particle surface area in bed}}{\text{Particles volume in bed}} \Big/ (1-\varepsilon) \\ &= \frac{\text{Particle surface area in bed}}{\text{Particles volume in bed}} (1-\varepsilon) = a_v (1-\varepsilon) \end{aligned} \tag{8}$$

The equivalent diameter of particles $D_p$ can be expressed by the specific surface area of particles $a_v$. If there are $N$ particles in the bed, the equivalent diameter of particles can be expressed by Eq. (9).

$$a_v = \frac{\text{Particles surface area in bed}}{\text{Particles volume in bed}} = \frac{N \pi D_p^2}{N \pi D_p^3 / 6} \Rightarrow D_p = \frac{6}{a_v} \tag{9}$$

Combining Eqs. (7)–(9), the hydraulic radius can be expressed as Eq. (10).

$$d = 4R = \frac{2 \varepsilon D_P}{3(1-\varepsilon)} \tag{10}$$

Then Eq. (9) is substituted for Eq. (6) to obtain Eq. (11).

$$\Delta P = \frac{E \mu \bar{v}}{2[\varepsilon D_p / 3(1-\varepsilon)]} = \frac{3 E \mu (1-\varepsilon) \bar{v}}{2 \varepsilon D_p} \tag{11}$$

The average velocity of fluid in porous media can be expressed by apparent velocity and porosity. The equation is

$$\bar{v} = \frac{v}{\varepsilon} \tag{12}$$

Eq. (12) is substituted for Eq. (11) to obtain Eq. (13).

$$\Delta P = \frac{3E\mu(1-\varepsilon)v}{2\varepsilon^2 D_p} \tag{13}$$

Because the model assumes that $l$ is approximately equal to average particle diameter $D_p$, the resistance loss per unit length is

$$\frac{\Delta P}{l} = \frac{3E\mu(1-\varepsilon)v}{2\varepsilon^2 D_p^2} \tag{14}$$

### 2.3 Darcy Origin Data

In 1856, Darcy conducted a series of sand seepage experiments in the fountain project in Dijon, France, and found Darcy's law. Table 1 shows the four sets of raw experimental data measured by Darcy on October 29 and 30, and December 6, 1855. The second and sixth columns in the table are flow rates, the third and seventh columns are experimentally measured total pressure drops, and the fourth and eighth columns are experimentally measured pressure drops per unit bed height. The calculation parameters in the following tables are as follows: the inner diameter of the bed section is 0.35 m, the porosity is 0.38, the particle diameter is 0.2 mm, the viscosity of water is 0.001 Pa.s, and the density of water is 1000 kg/m. It is necessary to mention that the original text of Darcy does not clearly specify the particle diameter. It only shows that the sand is filtered through a 0.77 mm sieve. It can only be inferred that the

**TABLE 1:** Darcy original data on pressure drop and discharge

| | Mean discharge (L/min) | Mean pressure (m) | Mean pressure (Pa/m) |
|---|---|---|---|
| First series (bed height 0.58 m) | 3.60 | 1.11 | 18,755.17 |
| | 7.65 | 2.36 | 39,875.86 |
| | 12.00 | 4.00 | 67,586.21 |
| | 14.28 | 4.90 | 82,793.10 |
| | 15.20 | 5.02 | 84,820.69 |
| | 21.80 | 7.63 | 128,920.69 |
| | 23.41 | 8.13 | 137,368.97 |
| | 24.50 | 8.58 | 144,972.41 |
| | 27.80 | 9.86 | 166,600.00 |
| | 29.40 | 10.89 | 184,003.45 |
| Second series (bed height 1.14 m) | 2.66 | 2.60 | 22,350.88 |
| | 4.28 | 4.70 | 40,403.51 |
| | 5.26 | 7.71 | 66,278.95 |
| | 8.60 | 10.34 | 88,887.72 |
| | 8.90 | 10.75 | 92,412.28 |
| | 10.40 | 12.34 | 106,080.70 |
| Third series (bed height 1.31 m) | 2.13 | 2.57 | 19,225.95 |
| | 3.90 | 5.09 | 38,077.86 |
| | 7.25 | 9.46 | 70,769.47 |
| | 8.55 | 12.35 | 92,389.31 |
| Fourth series (bed height 1.70 m) | 5.25 | 6.98 | 40,237.65 |
| | 7.00 | 9.95 | 57,358.82 |
| | 10.30 | 13.93 | 80,302.35 |

average particle size of the particles is less than 0.77 mm. As we know, the effect of particle diameter on drag calculation is generally inverse square ratio. It can be said that different particle diameter will eventually lead to a great difference between drag calculation and experimental value. Therefore, in order to determine the average diameter of the particles, we use the widely applicable Kozeny–Carman equation and the Ergun equation to try different particle diameters until the calculated values of the two equations are very close to the experimental values. The particle diameter was finally obtained to be 0.2 mm.

## 2.4 Determining $E$ in Eq. (14) and Comparison with Classical Formula

### *2.4.1 Determining $E$ in Eq. (14)*

The value of the constant $E$ can be calculated by substituting the velocity and

$$\frac{\Delta P}{l} = 300\frac{\mu(1-\varepsilon)v}{\varepsilon^2 D_p^2}\frac{\Delta P}{l} = 300\frac{\mu(1-\varepsilon)v}{\varepsilon^2 D_p^2}$$

unit pressure drop of Darcy origin data into Eq. (14). Since the first group of data of Darcy is less affected by external interference, we select them to calculate the $E$ value. The results are shown in Table 2. The column "Computed value" in Table 2 shows that the $E$ values range from 186.69 to 224.27, with an average of 205. Thus let $E$ equals 200, substitute the $E$ into Eq. (14) to obtain Eq. (15). The calculated unit pressure drop is in the column "Our equation value" of Table 2, and the error between the calculated and experimental values is in the seventh column "Error." It can be seen that the maximum error of Eq. (14) is 10.8%, which is smaller than that of the Kozeny–Carman equation and the Ergun equation, while the maximum error of the Ergun equation is 25.9%. Therefore, for the first set of experimental data of Darcy, the Eq. (14) derived in the paper is more accurate than the classical Kozeny–Carman equation and Ergun equation when the $E$ value is 200.

$$\frac{\Delta P}{l} = 300\frac{\mu(1-\varepsilon)v}{\varepsilon^2 D_p^2} \tag{15}$$

### *2.4.2 Further Verification and Comparison of Eq. (14)*

Equation (14), the Kozeny–Carman equation, and the Ergun equation are applied to the other three sets of data from Darcy. The calculation results and corresponding errors are shown in Table 3. From the error in Table 3, the calculation errors of the three equations are all too large, the maximum error is 63.7%, and the minimum error is also 27.2%.

**TABLE 2:** Determination of the average local resistance coefficient

| **Meanpressure (Pa/m)** | **Kozeny–Carman (Pa/m)** | | **Ergun equation (Pa/m)** | | **Our equation (Pa/m)** | | ***E* in our equation** | |
|---|---|---|---|---|---|---|---|---|
| Darcy experiment value | Equation value | Error | Equation value | Error | Our equation value | Error | Computed value | Average |
| 18,755.17 | 19,669.38 | 0.049 | 19,707.87 | 0.051 | 20,092.37 | 0.071 | 186.69 | The average value of $E$ is 205. Take $E$ equals 200. |
| 39,875.86 | 41,797.43 | 0.048 | 35,004.99 | 0.122 | 42,696.30 | 0.071 | 186.79 | |
| 67,586.21 | 65,564.59 | 0.030 | 55,064.82 | 0.185 | 66,974.58 | 0.009 | 201.83 | |
| 82,793.10 | 78,021.86 | 0.058 | 65,623.83 | 0.207 | 79,699.75 | 0.037 | 207.76 | |
| 84,820.69 | 83,048.48 | 0.021 | 69,893.22 | 0.176 | 84,834.47 | 0.000 | 199.97 | |
| 128,920.69 | 119,109.01 | 0.076 | 100,668.90 | 0.219 | 121,670.49 | 0.056 | 211.92 | |
| 137,368.97 | 127,905.59 | 0.069 | 108,215.55 | 0.212 | 130,656.25 | 0.049 | 210.28 | |
| 144,972.41 | 133,861.04 | 0.077 | 113,333.52 | 0.218 | 136,739.77 | 0.057 | 212.04 | |
| 166,600.00 | 151,891.30 | 0.088 | 128,871.31 | 0.226 | 155,157.78 | 0.069 | 214.75 | |
| 184,003.45 | 160,633.25 | 0.127 | 136,428.06 | 0.259 | 164,087.73 | 0.108 | 224.27 | |

**TABLE 3:** Three equations' prediction and their errors

| | Meanpressure (Pa/m) | Kozeny–Carman (Pa/m) | | Ergun equation (Pa/m) | | Our equation (Pa/m) | |
|---|---|---|---|---|---|---|---|
| | Darcy experiment value | Equation value | Error | Equation value | Error | Equation value | Error |
| Second series | 22,350.88 | 14,533.48 | 0.350 | 12,132.25 | 0.457 | 14,846.03 | 0.336 |
| | 40,403.51 | 23,384.70 | 0.421 | 19,541.66 | 0.516 | 23,887.60 | 0.409 |
| | 66,278.95 | 28,739.15 | 0.566 | 24,031.46 | 0.637 | 29,357.19 | 0.557 |
| | 88,887.72 | 46,987.96 | 0.471 | 39,376.28 | 0.557 | 47,998.45 | 0.460 |
| | 92,412.28 | 48,627.07 | 0.474 | 40,757.80 | 0.559 | 49,672.81 | 0.462 |
| | 106,080.70 | 56,822.65 | 0.464 | 47,673.42 | 0.551 | 58,044.64 | 0.453 |
| Third series | 19,225.95 | 11,637.71 | 0.395 | 9711.57 | 0.495 | 11,887.99 | 0.382 |
| | 38,077.86 | 21,308.49 | 0.440 | 17,802.25 | 0.532 | 21,766.74 | 0.428 |
| | 70,769.47 | 39,611.94 | 0.440 | 33,166.05 | 0.531 | 40,463.81 | 0.428 |
| | 92,389.31 | 46,714.77 | 0.494 | 39,146.08 | 0.576 | 47,719.39 | 0.483 |
| Fourth series | 40,237.65 | 28,684.51 | 0.287 | 23,985.61 | 0.404 | 29,301.38 | 0.272 |
| | 57,358.82 | 38,246.01 | 0.333 | 32,017.20 | 0.442 | 39,068.51 | 0.319 |
| | 80,302.35 | 56,276.27 | 0.299 | 47,211.97 | 0.412 | 57,486.52 | 0.284 |

It can be seen that the calculated values are smaller than the experimental values. The reason here can be attributed to the experiment itself. According to the original record, due to the influence of water hammer caused by the street fountain in the water supply system, the original paper clearly indicates that there are almost very strong oscillations when measuring these three sets of data. Despite this, it can be seen from the calculation results of the three equations that the prediction of Eq. (14) is still the best, followed by the Kozeny–Carman equation, and the Ergun equation is the worst.

In order to make up for the shortcomings of the last three groups of experiments, Charles Ritter carried out new tests on the above experimental devices on February 17 and 18, 1856. The test data are shown in Table 4. This data have little external influence. From the calculation results of the three equations, it can be seen that the error of Eq. (14) is the smallest, while the error of Kozeny–Carman equation is slightly larger. The Ergun equation errors are between 20% and 30%.

Therefore, it can be seen from this comparison that when the percolation Reynolds number of the porous medium is less than 1, both the Kozeny–Carman equation and Eq. (14) are applicable, but the Ergun equation has a large error.

**TABLE 4:** Comparison with experiment from Charles Ritter

| | Meanpressure (Pa/m) | Kozeny–Carman (Pa/m) | | Ergun equation (Pa/m) | | Our equation (Pa/m) | |
|---|---|---|---|---|---|---|---|
| | Darcy experiment value | Equation value | Error | Equation value | Error | Equation value | Error |
| Charles Ritter experiment on Feb. 17 and 18, 1856 | 116,530.91 | 102,717.86 | 0.119 | 86,647.88 | 0.256 | 104,926.84 | 0.100 |
| | 114,749.09 | 99,986.00 | 0.129 | 84,316.24 | 0.265 | 102,136.24 | 0.110 |
| | 112,076.36 | 98,346.89 | 0.123 | 82,917.97 | 0.260 | 100,461.87 | 0.104 |
| | 110,561.82 | 95,068.66 | 0.140 | 80,123.03 | 0.275 | 97,113.14 | 0.122 |
| | 109,670.91 | 98,893.26 | 0.098 | 83,384.00 | 0.240 | 101,019.99 | 0.079 |
| | 86,329.09 | 81,409.37 | 0.057 | 68,500.48 | 0.207 | 83,160.11 | 0.037 |
| | 75,192.73 | 66,110.96 | 0.121 | 55,527.28 | 0.262 | 67,532.70 | 0.102 |
| | 59,780.00 | 53,544.42 | 0.104 | 44,905.57 | 0.249 | 54,695.91 | 0.085 |
| | 51,494.55 | 43,163.36 | 0.162 | 36,154.81 | 0.298 | 44,091.60 | 0.144 |
| | 49,712.73 | 47,261.14 | 0.049 | 39,606.50 | 0.203 | 48,277.51 | 0.029 |
| | 26,549.09 | 24,586.72 | 0.074 | 20,549.07 | 0.226 | 25,115.47 | 0.054 |
| | 26,549.09 | 22,674.42 | 0.146 | 18,946.50 | 0.286 | 23,162.04 | 0.128 |

### 2.5 Verification of near Darcy Flow Regime

For providing a more general example we have added two sets of additional experimental data in the Darcy regime published by Bağcı et al. (2014) in the journal *Transport in Porous Media* in 2014. The experimental parameters are as follows: The inner diameter of the bed section is 51.4 mm. The diameter of the steel balls are 1.14 and 3.03 mm, respectively, and their corresponding porosities are 35.01% and 35.58%. The viscosity of water is 0.001 Pa.s, and the density of water is 1000 kg/m. In the following, only the speed and pressure drop experimental data of the near Darcy regime are selected for comparison.

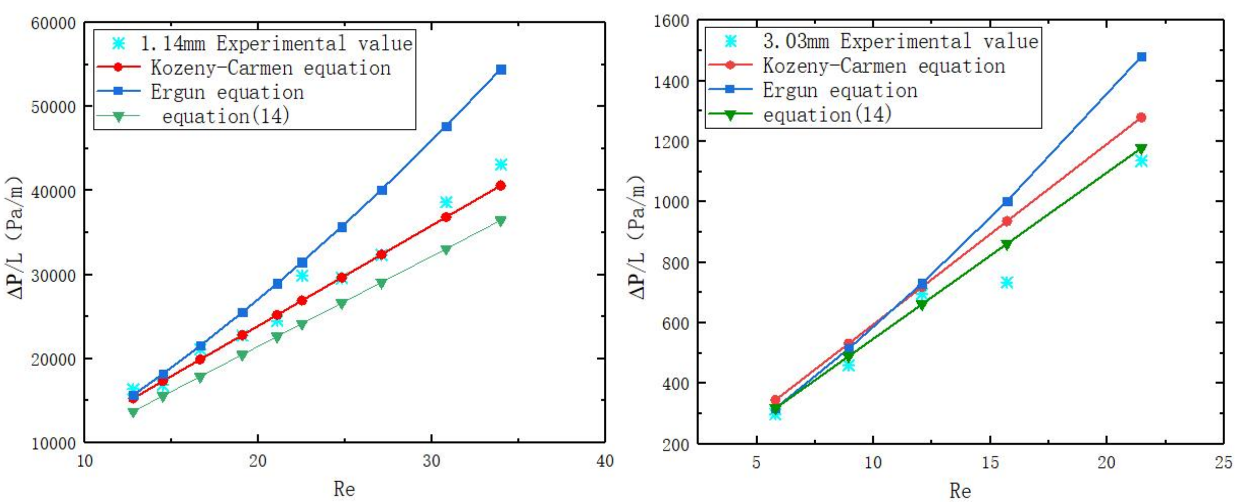
Figure 3 shows the compared result of the formulas with the two sets of experiment data (1.14 and 3.03 mm steel balls, respectively). Figure 3(a) is for the steel balls of 1.14 mm diameter, and Fig. 3(b) is for 3.03 mm diameter balls. In Fig. 3(a), the Ergun equation has the maximum deviation from experimental value and the Carman equation fit best with the experiment. Eq. (14) in the paper is in between, and the average error is 12.6%. In Fig. 3(b), Eq. (14) is 6% of average error and most consistent with the experimental value. The Carman equation is next, and its average error is 15%. Ergun's average error is 18%. Consequently, Eq. (14) has the better prediction of the two sets of experimental data when the Reynolds number is between 1 and 40. Combined with Darcy's experimental data, Eq. (14) may be applied to the flow in the natural sand and artificial porous media paced with small steel balls. Its Reynolds number can extend to 40. This indicates the minor resistance model based on pore throat unit is reasonable to some extent in quality and in quantity. However, the formula is derived from the simplified hypothesis of minor resistance, which is different from the real porous media. It needs lots of experimental data and other ways to verify its applicability in the future.

## 3. CONCLUSION

1. Based on the pore throat model and the local resistance hypothesis, a new formula for calculating the resistance of porous media is derived. The formula is as follows.

$$\frac{\Delta P}{l} = 300\frac{\mu(1-\varepsilon)v}{\varepsilon^2 D_p^2}$$

This formula is superior to the Kozeny–Carman equation and the Ergun equation for the experimental data from Darcy and Charles Ritter in the Darcy regime, as well as suitable to the ones of Bağcı et al. (2014) in the near Darcy flow regime. However, as a result of the simplifying assumption of minor resistance, the formula's




(a) (b)

**FIG. 3:** Comparison the formulas with two sets of experiment data: (a) 1.14 mm steel ball and (b) 3.03 mm steel ball

application needs to be tested with lots of experimental data and other ways such as numerical methods for the future studies.

2. Studies have shown that it is reasonable to assume that the resistance of porous media is the sum of many minor resistances, allowing a deeper understanding of the mechanism of internal resistance of porous media. At the same time, the average minor resistance coefficient of the porous medium was determined to be 200.

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